Towards a Statistical Network Calculus

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Motivation

Need simple analytical techniques to evaluate packet networks

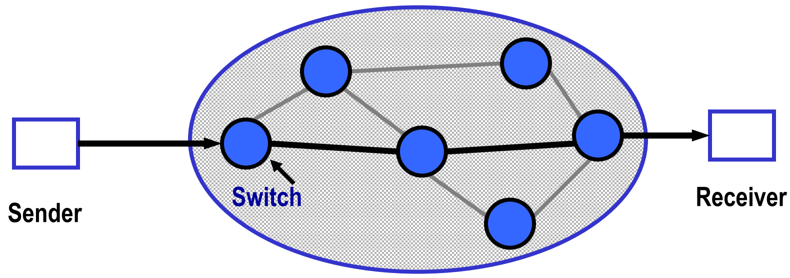
Queueing networks (Jackson, Kelly, BCMP):
→ limited to Poisson traffic
→ limited scheduling algorithms

Effective Bandwidth (Hui, Mitra, Kelly):
→ service guarantees
→ statistical multiplexing
→ wide variety of traffic (incl. LRD)
→ not well suited for scheduling

Deterministic calculus (Cruz):
→ service guarantees
→ no statistical multiplexing
→ worst-case traffic
→ scheduling

(Promise of) Statistical calculus:
→ service guarantees
→ statistical multiplexing
→ wide variety of traffic (incl. LRD)
→ scheduling

Delay Guarantees



- A **deterministic service** gives worst-case guarantees

$$\text{Delay} \leq d$$

- A **statistical service** provides probabilistic guarantees

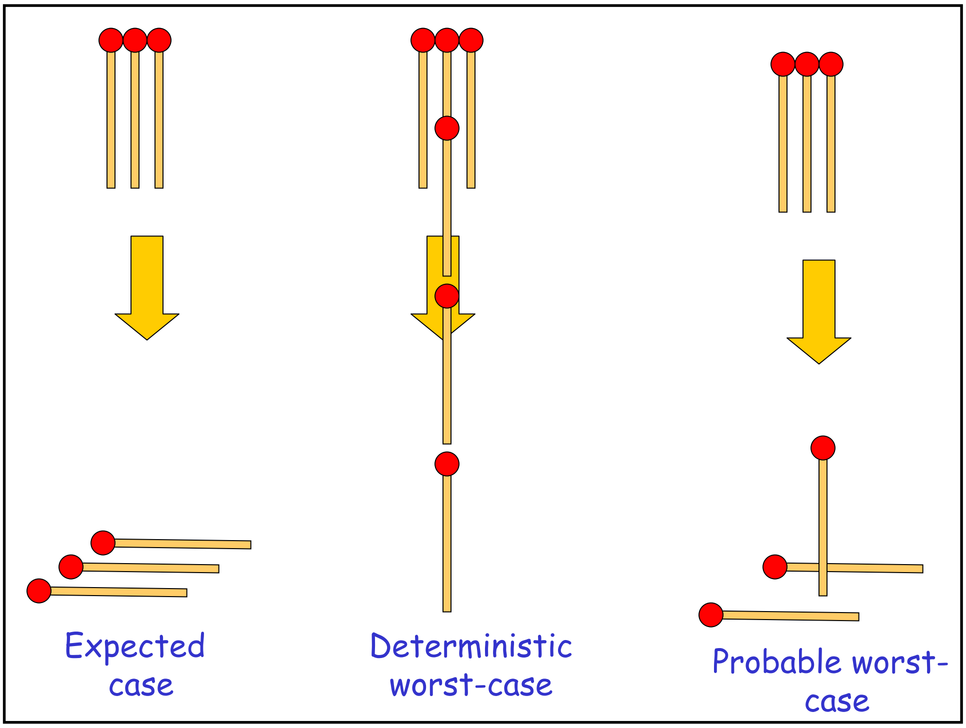
$$\Pr[\ \text{Delay} \geq d\ ] \leq \varepsilon$$

Multiplexing Gain

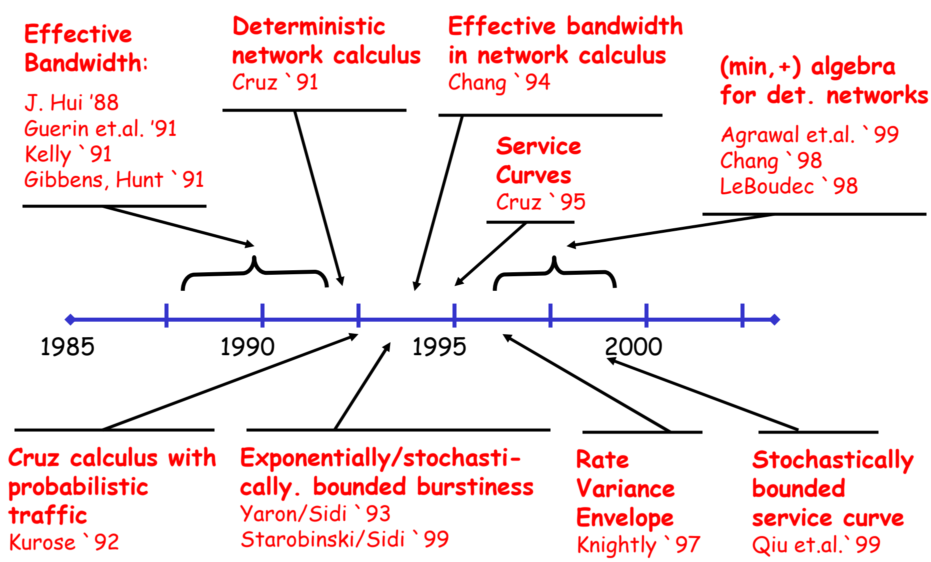
$$\begin{pmatrix}\text{Resources needed} \\ \text{to support} \\ \text{guarantees} \\ \text{for N flows}\end{pmatrix} << N \cdot \begin{pmatrix}\text{Resources needed} \\ \text{to support} \\ \text{guarantees} \\ \text{for 1 flow}\end{pmatrix}$$

Sources of multiplexing gain:

- Traffic Statistics
- Scheduling
- Statistical Multiplexing Gain



Related Work (small subset)



Our work

- R. Boorstyn, A. Burchard, J. Liebeherr, C. Oottamakorn. "Statistical Service Assurances for Packet Scheduling Algorithms", IEEE JSAC, Dec 2000.
  → **Effective envelope + scheduling**
- A. Burchard, J. Liebeherr, and S. D. Patek. "A Calculus for End-to-end Statistical Service Guarantees." Tech Report, May 2002.
  → **Effective service curve + (min,+) algebra**
- J. Liebeherr, A. Burchard, and S. D. Patek , "Statistical Per-Flow Service Bounds in a Network with Aggregate Provisioning", Infocom 2003.
  → **Application of effective service curves**
- C. Li, A. Burchard, J. Liebeherr, "Calculus with Effective Bandwidth", July 2002.
  → **Reconcile effective bandwidth and effective envelope**

Source Assumptions

Arrivals $A_j(t,t+\tau)$ from a flow j is a random process

Deterministic Calculus:

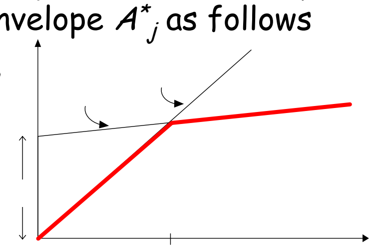
(A1) Additivity: For any $t_1 < t_2 < t_3$, we have:

$$A_j(t_1,t_2) + A_j(t_2,t_3) = A_j(t_1,t_3)$$

(A2) Subadditive Bounds: Traffic A_j is constrained by a subadditive deterministic envelope A^*_j as follows

$$A_j(t,t+\tau) \leq A^*_j(\tau) \quad ,\forall t, \forall \tau$$

with $\rho_j = \lim_{\tau \to \infty} A^*_j(\tau)/\tau$



Source Assumptions

Statistical Calculus:

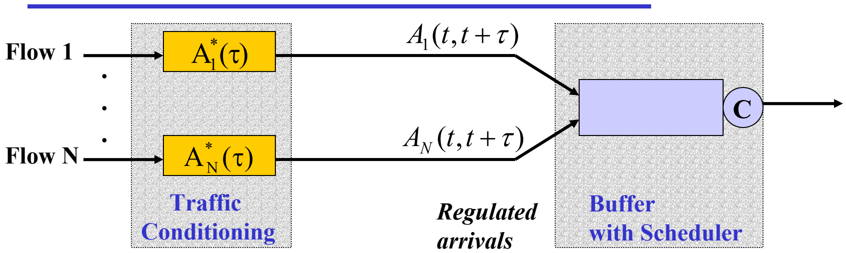
(A1) +(A2)

(A3) Stationarity: The A_j are stationary random processes

(A4) Independence: The A_i and A_j ($i \neq j$) are stochastically independent

(No assumptions on arrival distribution!)

Aggregating Arrivals



Arrivals from multiple flows: $A_C = \sum_j A_j$

Deterministic Calculus:

Worst-case of multiple flows is sum of the worst-case of each flow

$$A_C(t, t+\tau) \leq \sum_j A_j^*(\tau)$$

Aggregating Arrivals

Statistical Calculus:
"Effective Envelope" → bounds aggregate of flows with high probability

- **effective envelope** $\mathcal{G}_{\mathcal{C}}^{\varepsilon}$:

$$Pr\{A_{\mathcal{C}}(t, t+\tau) \leq \mathcal{G}_{\mathcal{C}}^{\varepsilon}(\tau)\} \geq 1-\varepsilon \quad \forall t, \tau$$

- **strong effective envelope** $\mathcal{H}_{\mathcal{C}}^{\ell,\varepsilon}$:

$$Pr\{\forall [t, t+\tau] \subseteq I_{\ell} : A_{\mathcal{C}}(t, t+\tau) \leq \mathcal{H}_{\mathcal{C}}^{\ell,\varepsilon}(\tau)\} \geq 1-\varepsilon, \forall I_{\ell}$$

Effective envelopes are non-random functions

Obtaining Effective Envelopes

$$\mathcal{G}_{\mathcal{C}}^{\varepsilon}(t) = \inf_{s>0} \frac{1}{s}\Big(\sum_{j\in\mathcal{C}} \log \overline{M}_j(s,t) - \log \varepsilon\Big)$$

with $\overline{M}_j(s,t) = 1 + \frac{\rho_j t}{A_j^*(t)}\left(e^{sA_j^*(t)} - 1\right)$

$$\mathcal{H}_{\mathcal{C}}^{\ell,\varepsilon'}(t) \leq \mathcal{G}_{\mathcal{C}}^{\varepsilon}(\gamma t + a), \qquad 0 \leq t \leq \ell$$

with $\varepsilon' \leq \varepsilon \cdot \frac{\ell}{a} \frac{\sqrt{\gamma}+1}{\sqrt{\gamma}-1}$

$a \in (0, \ell)$

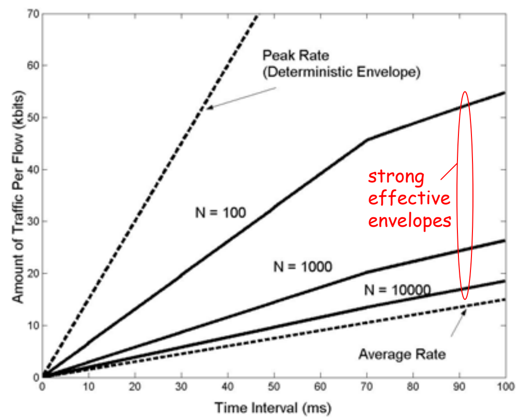
$\gamma > 1$

Effective Envelope vs. Deterministic Envelopes

A*=min (Pt, σ+ρt)

Type 1 flows:
P =1.5 Mbps
ρ = .15 Mbps
σ =95400 bits

Type 2 flows:
P = 6 Mbps
ρ = .15 Mbps
σ = 10345 bits



Type 1 flows

Scheduling Algorithms (here EDF)

- EDF scheduler with rate 1 that servers Q classes
- Class-q arrival has delay bound d_q

Deterministic service:

$$\sup_{\hat{\tau}} \left\{ \sum_p A^*_{\mathcal{C}_p}(\tau_p + \hat{\tau}) - \hat{\tau} \right\} \leq d_q$$

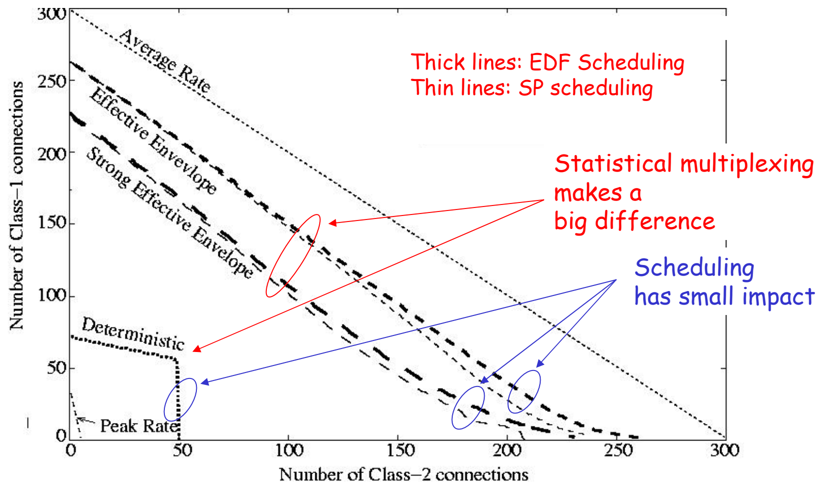
Statistical service:

$$\sup_{\hat{\tau}} \left\{ \sum_p \mathcal{H}^{\ell,\varepsilon/Q}_{\mathcal{C}_p}(\tau_p + \hat{\tau}) - \hat{\tau} \right\} \leq d_q$$

$$\tau_p = \max\{-\hat{\tau}, d_q - d_p\}.$$

Effective Envelope vs. Deterministic Envelope

C= 45 Mbps, $\varepsilon = 10^{-6}$
Delay bounds: Type 1: d_1=100 ms, Type 2: d_2=10 ms,



Effective Envelopes and Effective Bandwidth

Effective Bandwidth (Kelly, Chang)

$$\alpha(s,\tau) = \sup_{t\geq 0} \left\{ \frac{1}{s\tau} \log E[e^{s(A[t+\tau]-A[t])}] \right\}$$

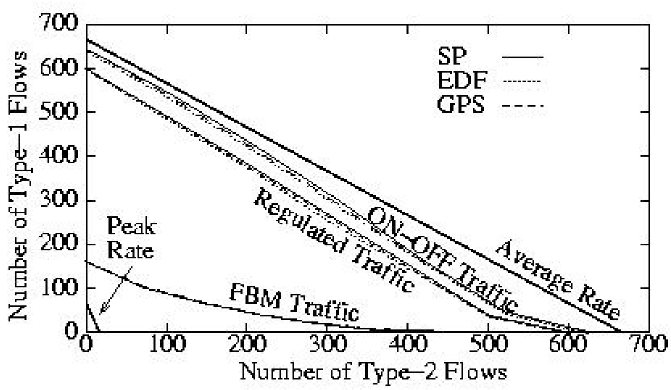
$$s, \tau \in (0, \infty)$$

Given $\alpha(s,\tau)$, an effective envelope is given by

$$\mathcal{G}^{\varepsilon}(\tau) = \inf_{s>0} \{ \tau\alpha(s,\tau) - \frac{\log \varepsilon}{s} \}$$

Effective Envelopes and Effective Bandwidth

Now, we can calculate statistical service guarantees for schedulers and traffic types



Schedulers:

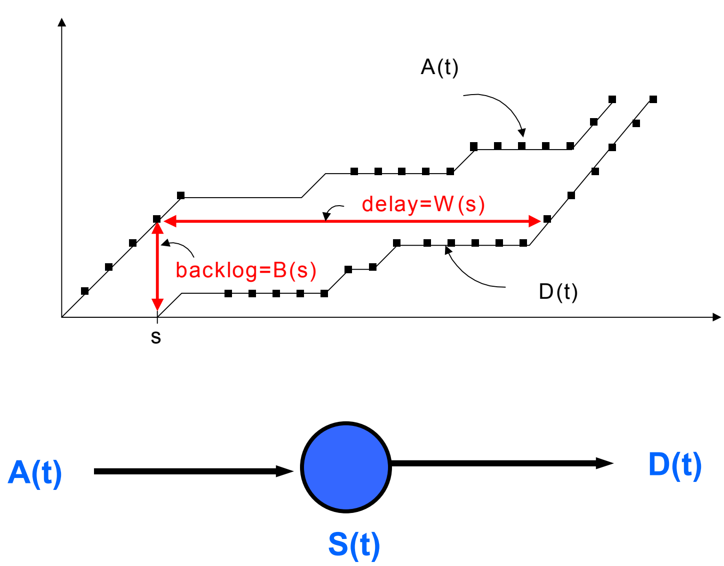
SP- Static Priority
EDF – Earliest Deadline First
GPS – Generalized Processor Sharing

Traffic:

Regulated – leaky bucket
On-Off – On-off source
FBM – Fractional Brownian Motion

C= 100 Mbps, ε = 10^{-6}

Network Calculus with Min-Plus Algebra



Convolution and Deconvolution operators

- **Convolution operation:**

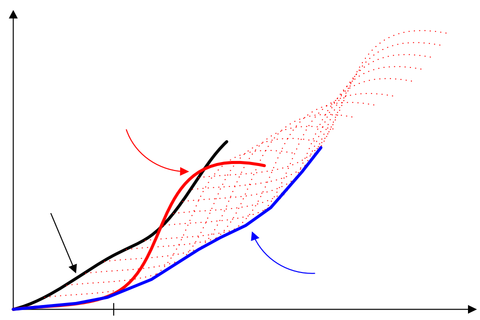
$$f * g(t) = \inf_{\tau \in [0,t)} f(t-\tau) + g(\tau)$$

- **Deconvolution operation**

$$f \oslash g(t) = \sup_{\tau \in [0,t)} f(t+\tau) - g(\tau)$$

- **Impulse function:**

$$\delta_T(t) = \begin{cases} \infty & ,t > \tau \\ 0 & ,t \le \tau \end{cases}$$



Deterministic Network Calculus

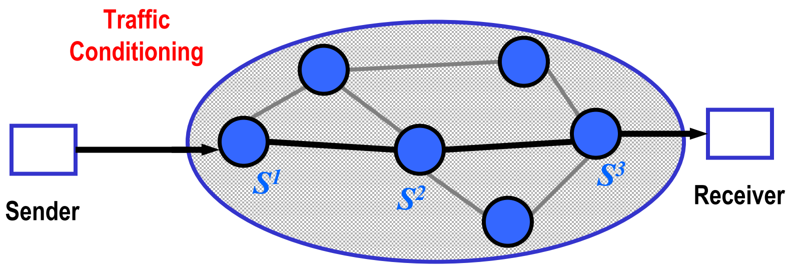
Cruz \`95:
A (minimum) service curve for a flow is a function S such that: $D(t) \ge A * S(t), \quad \forall t \ge 0$

(min,+) results (Cruz, Chang, LeBoudec)

1. Output Envelope: $A^* \oslash S$ is an envelope for the departures
2. Backlog bound: $A^* \oslash S(0)$ is an upper bound for the backlog
3. Delay bound: An upper bound for the delay is

$$d_{max} = \inf \{d \ge 0 \mid \forall t \ge 0 : A^*(t-d) \le S(t)\}$$

Network Service Curve (Cruz, Chang, LeBoudec)



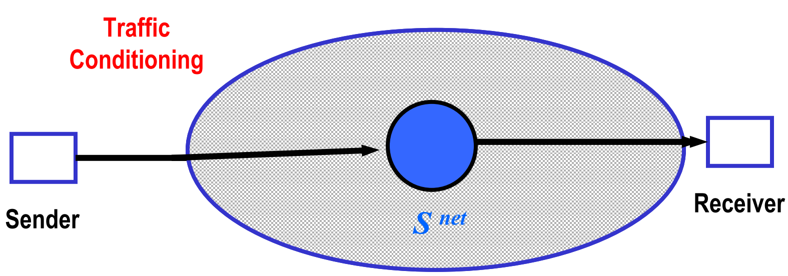
Network Service Curve:

If S^1, S^2 and S^3 are service curves for a flow at nodes, then

$$S^{net} = S^1 * S^2 * S^3$$

is a service curve for the entire network.

Network Service Curve (Cruz, Chang, LeBoudec)



Network Service Curve:

If S^1, S^2 and S^3 are service curves for a flow at nodes, then

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Statistical Network Calculus

A (minimum) effective service curve for a flow is a function S^{ε} such that:

$$Pr[D(t) \geq A * \mathcal{S}^{\varepsilon}(t)] \geq 1 - \varepsilon, \quad \forall t \geq 0$$

(min,+) results

1. Output Envelope: $A^* \oslash S^{\varepsilon}$ is an envelope for the departures **with probability** ε
2. Backlog bound: $A^* \oslash S^{\varepsilon}(0)$ is an upper bound for the backlog with **probability** ε
3. Delay bound: An upper bound for the delay **with probability** ε is $\inf\{d \geq 0 \mid \forall t \geq 0 : A^*(t-d) \leq S^{\varepsilon}(t)\}$

Effective Network Service Curve

Network Service Curve:

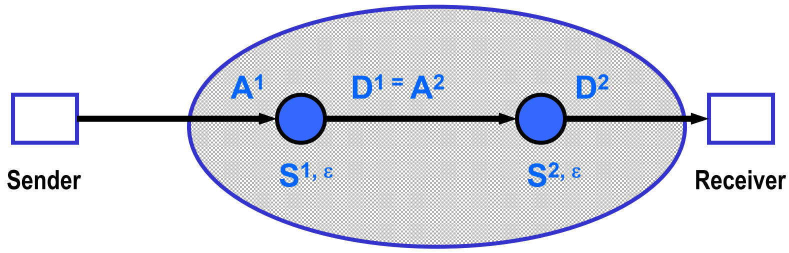
If $\mathcal{S}^{1,\varepsilon}, \mathcal{S}^{2,\varepsilon}, \ldots, \mathcal{S}^{H,\varepsilon}$ are effective service curves for a flow, then for all $t \geq 0$:

$$Pr\{D(t) \geq A*(\mathcal{S}^{1,\varepsilon}*\ldots*\mathcal{S}^{H,\varepsilon}*\delta_{Ha})(t)\} \geq 1-\varepsilon H\frac{t}{a}$$

Unfortunately, this network service is not very useful!

A "good" network service curve can be obtained by working with a modified service curve definition (see technical report)

What is the cause of the problem with the network effective service curve?



In the convolution

$$D^2(t) \geq A * \mathcal{S}^{2,\varepsilon}(t) = \inf_{\tau \in [0,t]} A^2(t-\tau) + \mathcal{S}^{2,\varepsilon}(\tau)$$

the range [0,t] where the infimum is taken is a random variable that does not have an a priori bound.

Statistical calculus: Summary

- Statistical network calculus = Deterministic network calculus + large deviations
- Statistical calculus preserves much (but not all) of the deterministic calculus:
  -

Conclusions

- Statistical network calculus = Deterministic network calculus + large deviations
- Statistical calculus preserves much (but not all) of the deterministic calculus
  - Consideration of scheduling
  - Single node (min, +) calculus results
- … and in addition:
  - exploits statistical multiplexing
  - deals with various traffic types
- Constructs of effective envelope and effective service curves are useful (at least to us)

Open Issues

- When does the statistical network service curve become simple? (starting point: CS-02-19, technical report)
- Which set of assumptions (if any) lead to a product-form-network analogue in the network calculus?
- Most important: Computational algorithms

  There are not yet any (simple) computational techniques for a statistical network calculus